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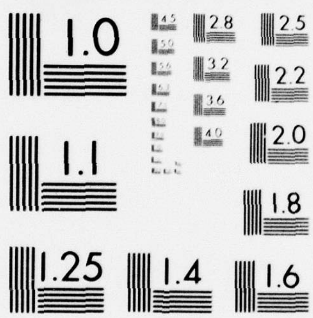

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Three typical examples illustrate the use of operations research in electronic warfare and are of considerable interest in themselves. The first example concerns radar decoys and uses Markov chains in the analysis. The second example concerns jamming strategies and employs game theory. The final example concerns resource allocation for an aircraft attack and involves both game theory and optimization methods.

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1. INTRODUCTION

The methods of operations research have been extensively used in military applications. This paper presents three typical examples that illustrate the use of operations research in electronic warfare and are of considerable interest in themselves. The first example concerns radar decoys and uses Markov chains in the analysis. It is shown how to calculate two quantities of fundamental importance in the evaluation of the defensive capability of a radar-decoy array. These quantities are the probability of radar survival after an antiradiation missile (ARM) attack and the expected number of ARM's required to destroy the radar. The second example concerns jamming strategies and employs game theory. It is concluded that, subject to certain restrictions, wideband barrage jamming and frequency agility are the optimal strategies for a jammer and a radar, respectively. The final example concerns resource allocation for an aircraft attack and involves both game theory and optimization methods. A methodology is developed for the rational allocation of resources when there is uncertainty about the defense's capabilities. *Note: The notation in each section is independent of the notations in the other sections.*

2. DECOY PROBLEM

An ARM is a missile with passive homing devices that enable it to seek a source of electromagnetic radiation of certain frequencies. Radar stations are prime possibilities as ARM targets. To attack a radar station, an aircraft conveying one or more ARM's may attempt to approach the radar at a low altitude to avoid detection. When the aircraft is sufficiently close to the radar, it rapidly ascends to a high altitude while launching the ARM toward the radar, as illustrated in figure 1. If an ARM attack is known to be imminent, the radar transmissions should be curtailed as much as possible. However, if the

radar is part of an air defense system, it may not be feasible, in view of the surveillance requirements of an air defense system, to curtail radar transmissions sufficiently to thwart the ARM homing device. In this case, a possible countermeasure to the ARM threat is to deploy decoys in the vicinity of the radar.

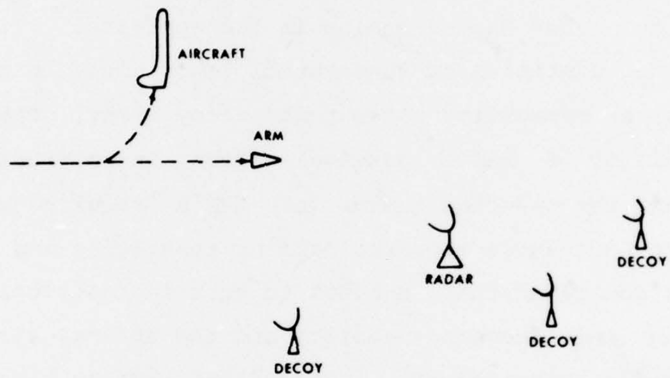

Figure 1. Antiradiation missile (ARM) attack against radar-decoy array.

It is important to know the probability of radar destruction when N identical decoys are deployed around a radar and M ARM's are launched against the radar. If the ARM's are launched sequentially, the problem can be formulated as a finite Markov chain.[1,2]

Let X_n be a random variable representing the state of the radar-decoy array after n attacks by single ARM's. The states of the chain, which are the possible values that X_n may assume, are labeled by the integers $i = 0, 1, \ldots, N + 1$. The Markov assumption implies the probability that $X_n = i$ is dependent on the value of X_{n-1}, but not

[1] *F. S. Hillier and G. J. Lieberman, Operations Research, 2nd ed., Holden-Day, San Francisco (1974).*

[2] *S. Karlin and H. M. Taylor, A First Course in Stochastic Processes, Academic Press, New York (1975).*

also on the values of $X_0, X_1, \ldots, X_{n-2}$. The radar system is defined to be in state i for $i = 0, 1, \ldots, N$ if $N - i$ decoys and the radar itself remain intact. The radar system is defined to be in state $N + 1$ if the radar has been destroyed by the ARM attacks, regardless of how many decoys are intact. In the terminology of Markov chains, state $N + 1$ is an absorbing state, whereas the other states are transient.

An alternative formulation of the problem is to assign separate states for different numbers of surviving decoys when the radar has been destroyed. However, this more detailed formulation seems unnecessary for most applications, so it is not pursued in this paper.

Each attack by an ARM defines a transition by the radar system from one state to the same or another state. In most applications, the transition probabilities of the Markov chain can be assumed to be stationary. Accordingly, the elements of the transition matrix $\underline{P}$ are labeled P_{ij}, where i denotes the row and j denotes the column. The element P_{ij} represents the probability that $X_n = j$, given that $X_{n-1} = i$. We define $\underline{Z}_m$ as a row vector with element i having a value equal to the probability that $X_m = i$. For example, the first element (element zero) of $\underline{Z}_m$ denotes the probability that $X_m = 0$, that is, the probability that the radar-decoy array is completely intact after m sequential ARM attacks. Thus, $\underline{Z}_{m+1} = \underline{Z}_m\underline{P}$. The elements of the n-step transition matrix, $\underline{P}^n$, are labeled P_{ij}^n for $n > 1$. By definition, $\underline{Z}_{m+n} = \underline{Z}_m\underline{P}^n$. The probability of radar destruction after M ARM attacks is given by the element $P_{0,N+1}^M$. Since ARM attacks cannot increase the number of decoys, $P_{ij} = 0$ for $j < i$.

An attack by a single ARM can be modeled as consisting of two phases. During the first phase, the ARM selects a target from among the radar and the decoys. If the radar system is in state i, then r_i

denotes the probability that the radar is selected; it follows that $1 - r_i$ is the probability that one of the decoys is selected. During the second phase, the ARM attacks its chosen target. Thus, α_i represents the probability of destruction if the radar has been selected, and β_i represents the probability of destruction if one of the decoys has been selected. These quantities are often called single-shot kill probabilities. Since it is much easier to harden a decoy than a radar against a detonation, we have $\beta_i \leq \alpha_i$ in all practical cases. If the radar system is in state N, no decoys remain, so $r_N = 1$.

The preceding definitions imply that the diagonal elements of the transition matrix are given by

$$P_{ii} = 1 - r_i\alpha_i - \left(1 - r_i\right)\beta_i \ , \quad i = 0, 1, \ldots, N-1 \ ,$$

$$P_{N,N} = 1 - \alpha_N \ , \tag{1}$$

$$P_{N+1,N+1} = 1 \ .$$

The transition matrix has the form

$$\underline{P} = \begin{bmatrix} P_{00} & \left(1 - r_0\right)\beta_0 & 0 & \cdots & 0 & r_0\alpha_0 \\ 0 & P_{11} & \left(1 - r_1\right)\beta_1 & \cdots & 0 & r_1\alpha_1 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 1 - \alpha_N & \alpha_N \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{2}$$

It follows that

$$P_{ii}^{n} = \left(P_{ii}\right)^{n} \ . \tag{3}$$

If the decoys are well designed, the ARM will not be able to distinguish between the decoys and the radar. In this case, the target selection can be considered random, with each of the N - i + 1 possible targets equally likely to be chosen. Thus,

$$r_i = \frac{1}{N - i + 1} , \quad i = 0, 1, \ldots, N . \tag{4}$$

The indistinguishability assumption, if not warranted, still can be used to calculate a lower bound on the probability of radar destruction.

As a simple example, suppose N = 2 decoys are deployed and M = 3 ARM's are sequentially launched against a radar-decoy array of indistinguishable targets. It is assumed that once a target is selected by an ARM, the probability of its destruction is independent of the state; that is, $\alpha_i = \alpha$ and $\beta_i = \beta$ for all i. The transition matrix is

$$\underline{P} = \begin{bmatrix} \frac{3 - \alpha - 2\beta}{3} & \frac{2\beta}{3} & 0 & \frac{\alpha}{3} \\ 0 & \frac{2 - \alpha - \beta}{2} & \frac{\beta}{2} & \frac{\alpha}{2} \\ 0 & 0 & 1 - \alpha & \alpha \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{5}$$

By means of matrix multiplication, we can obtain $\underline{P}^3$, which gives the transition probabilities relative to the termination of the attack. Of particular interest is the element $P^3_{0,3}$, which appears in the upper right corner of the $\underline{P}^3$ matrix and corresponds to the probability of radar destruction after three ARM attacks. Carrying out the algebra and denoting this element by γ, we obtain

$$\gamma = \alpha + \left[\frac{(\alpha - \beta)\alpha(2\alpha - 5\beta - 18)}{54}\right] . \tag{6}$$

Since $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$, it is apparent that

$$\gamma < \alpha \quad \text{if } \beta < \alpha,$$

$$\gamma = \alpha \quad \text{if } \beta = \alpha, \qquad (7)$$

$$\gamma > \alpha \quad \text{if } \beta > \alpha .$$

To put this result in perspective, consider the simultaneous launching of three ARM's against the radar-decoy array. If it is possible to assign each ARM to a different target out of the three, then an ARM will be launched against the radar with certainty. Thus, the probability of radar destruction after the attack is equal to the single-shot kill probability for the radar; that is, $\gamma = \alpha$ in this case. We conclude that the simultaneous attack of each radiator in the array, if feasible, is advantageous to the attacker when $\beta < \alpha$ and the radiators are indistinguishable to an ARM.

In evaluating the defensive capability of a radar-decoy array against an ARM attack, there are two quantities of primary interest. One quantity, which was evaluated in equation (7), is the probability of radar destruction after the attack. The second quantity is the expected number of ARM's required to destroy the radar. We next show how to calculate the latter quantity when the ARM's are launched sequentially.

It is intuitively clear and can be shown mathematically from equation (2) that the probability of eventual radar destruction is unity if $P_{ii} < 1$, $i \neq N + 1$, and an unlimited number of ARM's are available. (Karlin and Taylor, pp. 90-91, give the mathematical details of this type of calculation.[2]) Let $f^{n}_{i,N+1}$ denote the probability that, starting from state i, radar destruction occurs during the nth transition. It follows that

[2] *S. Karlin and H. M. Taylor, A First Course in Stochastic Processes, Academic Press, New York (1975).*

$$\sum_{n=1}^{\infty} f^n_{i,N+1} = 1 \ , \quad i = 0, 1, \ldots, N \ , \tag{8}$$

and

$$f^{n+1}_{i,N+1} = \sum_{k=0}^{N} P_{ik} f^n_{k,N+1} \ , \ n \geq 1 \ , \quad i = 0, 1, \ldots, N \ . \tag{9}$$

By definition, the expected number of ARM's required to destroy the radar, if the initial state is i, is

$$\mu_{i,N+1} = \sum_{n=1}^{\infty} n f^n_{i,N+1} \ , \quad i = 0, 1, \ldots, N \ . \tag{10}$$

Multiplying equation (10) by P_{ki} and summing over i, we obtain

$$\sum_{i=0}^{N} P_{ki} \mu_{i,N+1} = \sum_{n=1}^{\infty} n \sum_{i=0}^{N} P_{ki} f^n_{i,N+1} \ . \tag{11}$$

Using equations (8) through (10) in equation (11), we derive

$$\mu_{k,N+1} = 1 + \sum_{i=0}^{N} P_{ki} \mu_{i,N+1} \ , \quad k = 0, 1, \ldots, N \ . \tag{12}$$

To obtain the quantity of interest, $\mu_{0,N+1}$, these N + 1 equations must be solved simultaneously. Since $P_{ij} = 0$ when $j < i$, it follows from linear algebra or matrix theory that a solution exists if $P_{ii} < 1$, $i \neq N + 1$.

To illustrate the use of equation (12), we return to the example considered previously in which N = 2. We may write

$$\begin{aligned} \mu_{23} &= 1 + P_{22}\mu_{23} \ , \\ \mu_{13} &= 1 + P_{11}\mu_{13} + P_{12}\mu_{23} \ , \\ \mu_{03} &= 1 + P_{00}\mu_{03} + P_{01}\mu_{13} + P_{02}\mu_{23} \ . \end{aligned} \tag{13}$$

Solving these equations simultaneously and substituting from equation (5), we obtain

$$\mu_{03} = \frac{2}{\alpha} + \frac{(\alpha - \beta)}{\alpha(\alpha + \beta)} , \tag{14}$$

assuming that $\alpha > 0$. If the costs required to ensure various values of α and β are known, this equation can be used to determine μ_{03} as a function of cost. The quantity μ_{03} is a measure of the defensive capability of the radar and two decoys. Similar calculations can be done to assess the relationship between the cost and the defensive capability of a radar and N decoys.

3. JAMMING OF RADAR

The duel between a radar and a jammer can be analyzed by means of game theory if suitable restrictions are placed on the options available to the combatants. As is usual in game theory, it is assumed that both combatants choose their strategies simultaneously without any knowledge of each other's choices.

We define a basic time interval that is equal to the round-trip time of one or more radar pulses. During each time interval, the radar may transmit the pulse in one of n bands of equal bandwidth. A pure strategy consists of choosing a single fixed band for transmission during all time intervals of operation. A mixed strategy entails the randomized selection of a band during each basic time interval. The jammer is assumed to employ noise jamming of fixed average power. A pure strategy for the jammer consists of jamming a fixed subset of the n radar bands with uniformly distributed jamming energy during all time intervals of operation. A mixed strategy entails the randomized selection of a subset of bands to be jammed during each basic time interval. Within the jammed bands, the jamming energy is uniformly

distributed with a power spectral density denoted by J_0. If a single band is jammed, then J_0 has its maximum possible value J_1. If k bands are simultaneously jammed, then $J_0 = J_1/k$ in each of these bands. Random thermal noise of power spectral density N_0 is assumed to be present at the radar receiver.

We construct a payoff matrix with the rows corresponding to the radar's pure strategies and the columns corresponding to the jammer's pure strategies. Thus, the payoff matrix has n rows and $2^n - 1$ columns. The payoff is defined to be the signal-to-interference ratio at the output of the radar receiver. We assume that the radar receiver contains a matched filter and that the jamming can be modeled as independent, band-limited, white Gaussian noise. Consequently, the signal-to-interference ratio at the receiver output is $2E/(J_0 + N_0)$, where E is the pulse energy at the receiver output.[3] Although it may not always be an adequate measure of radar performance, the signal-to-interference ratio is used for the payoff in order to simplify the mathematical analysis.

Although the jammer has $2^n - 1$ possible pure strategies, there are only n distinct classes of strategies. A specific class is defined according to how many bands are jammed, as indicated in table I. If a column of the payoff matrix is associated with a strategy belonging to class C_k, then the entries in the column have the two possible values listed in table I. The form of the payoff matrix is illustrated in table II.

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[3]*A. Whalen, Detection of Signals in Noise, Academic Press, New York (1971).*

TABLE I. POSSIBLE JAMMER STRATEGIES

Class	Definition	Number of pure strategies in class	Entries in corresponding columns of payoff matrix
C_1	$J_0 = J_1$ in one band; other bands not jammed	n	1 entry: $\frac{2E}{J_1 + N_0}$ n - 1 entries: $\frac{2E}{N_0}$
C_k	$J_0 = J_1/k$ in k bands; other bands not jammed	$\begin{bmatrix} n \\ k \end{bmatrix}$	k entries: $\frac{2E}{\frac{J_1}{k} + N_0}$ n - k entries: $\frac{2E}{N_0}$
C_n	$J_0 = J_1/n$ in all bands	1	All entries: $\frac{2E}{\frac{J_1}{n} + N_0}$

Note: J_0 = jamming power spectral density within a jammed band.
J_1 = maximum possible value of J_0.
k = number of jammed bands.
n = number of bands.
E = pulse energy at the receiver output.
N_0 = power spectral density of thermal noise.

TABLE II. FORM OF PAYOFF MATRIX

Jammer strategies

Radar strategies

$\frac{2E}{J_1 + N_0}$	$\frac{2E}{N_0}$	. . .	$\frac{2E}{\frac{J_1}{k} + N_0}$	. . .	$\frac{2E}{\frac{J_1}{n} + N_0}$
$\frac{2E}{N_0}$	$\frac{2E}{J_1 + N_0}$	. . .	$\frac{2E}{N_0}$	. . .	$\frac{2E}{\frac{J_1}{n} + N_0}$
$\frac{2E}{N_0}$	$\frac{2E}{N_0}$	. . .	$\frac{2E}{\frac{J_1}{k} + N_0}$	. . .	$\frac{2E}{\frac{J_1}{n} + N_0}$
$\vdots$	$\vdots$	. . . .	$\vdots$	. . . .	$\vdots$
$\frac{2E}{N_0}$	$\frac{2E}{N_0}$	. . .	$\frac{2E}{N_0}$	. . .	$\frac{2E}{\frac{J_1}{n} + N_0}$

Note: E = pulse energy at the receiver output.

J_1 = maximum possible value of J_0.

N_0 = power spectral density of thermal noise.

k = number of jammed bands.

n = number of bands.

The upper value of the game is the maximum payoff that can result if the jammer selects the pure strategy resulting in the smallest maximum payoff. From tables I and II, it is seen that this strategy is the unique strategy of class C_n. Thus, the upper value of the game is

$$V_1 = \frac{2E}{\frac{J_1}{n} + N_0} . \tag{15}$$

The lower value of the game is the minimum payoff that can result if the radar selects the pure strategy resulting in the largest minimum payoff. From the tables, it is seen that all pure strategies have the same minimum payoff. Thus, the lower value of the game is

$$V_2 = \frac{2E}{J_1 + N_0} . \tag{16}$$

A saddle point is defined as an entry that is both the minimum in its row and the maximum in its column. According to game theory, the existence of a saddle point implies that pure strategies are optimal for both combatants. Since the upper and lower values of this game are not equal, a saddle point does not exist for this game if $n > 1$. Thus, a mixed strategy is optimal for one or both of the combatants.[1,4]

Each row of the payoff matrix has the same array of entry values, although the ordering differs from row to row. Because of this symmetry, it is intuitively plausible that an optimal strategy for the radar operator is to choose a band randomly, with each band having a probability of being chosen equal to $1/n$. A radar that is operated in this mode is called a frequency-agile radar.

Let $\underline{p}$ be a vector with entries denoting the probabilities of the radar's strategies; let $\underline{q}$ be a vector with entries denoting the

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[1] *F. S. Hillier and G. J. Lieberman, Operations Research, 2nd ed., Holden-Day, San Francisco (1974).*

[4] *D. J. White, Fundamentals of Decision Theory, American Elsevier, New York (1976).*

probabilities of the jammer's strategies. We represent the payoff matrix by $\underline{A}$. The expected payoff is

$$\bar{V}(\underline{p},\underline{q}) = \underline{p}\,\underline{A}\,\underline{q} = \sum_{i=1}^{n} \sum_{j=1}^{2^n-1} p_i A_{ij} q_j \; . \tag{17}$$

We denote by $\underline{p}_1$ the particular vector corresponding to a frequency-agile radar; that is, the elements of $\underline{p}_1$ are given by $p_{1i} = 1/n$, for all i. It follows that

$$\bar{V}(\underline{p}_1,\underline{q}) = \frac{1}{n} \sum_{j=1}^{2^n-1} q_j \sum_{i=1}^{n} A_{ij} \; . \tag{18}$$

From the tables, if j belongs to class C_k,

$$\sum_{i=1}^{n} A_{ij} = \frac{2Ek}{\frac{J_1}{k} + N_0} + \frac{2E(n-k)}{N_0} \; , \quad j \in C_k \; . \tag{19}$$

It is easy to verify that a unique minimum value of equation (19) results when $j \in C_n$. Thus, with $\underline{q}_1$ corresponding to the pure strategy of jamming all bands, we conclude that, for any $\underline{q}$,

$$\bar{V}(\underline{p}_1,\underline{q}_1) \leq \bar{V}(\underline{p}_1,\underline{q}) \; . \tag{20}$$

Since all the elements of $\underline{q}_1$ are zero except the element corresponding to strategy C_n, it is straightforward to verify, by using equation (17), that

$$\bar{V}(\underline{p},\underline{q}_1) = \bar{V}(\underline{p}_1,\underline{q}_1) \tag{21}$$

for all $\underline{p}$. Combining equations (20) and (21), we have

$$\overline{V}(\underline{p},\underline{q}_1) \le \overline{V}(\underline{p}_1,\underline{q}_1) \le \overline{V}(\underline{p}_1,\underline{q}) \; . \tag{22}$$

It is known from the theory of games[4] that this relation establishes $\underline{p}_1$ and $\underline{q}_1$ as the vectors corresponding to the optimal strategies for the radar and the jammer, respectively. The value of the game, determined from equations (18) and (19), is

$$\overline{V}(\underline{p}_1,\underline{q}_1) = \frac{2E}{\frac{J_1}{n} + N_0} \; . \tag{23}$$

This quantity is the signal-to-interference ratio that results when both combatants employ optimal strategies.

We have shown that the jammer should spread the jamming energy uniformly throughout all n bands. If the bands are contiguous, we conclude that wideband barrage jamming is preferable to narrowband spot jamming. The signal-to-interference ratio at the output of the radar receiver is given by equation (23) when barrage jamming is employed.

Although we have obtained an interesting and useful result, we have rather artificially restricted the available strategies. By insisting that the game strategies be chosen simultaneously, we have excluded the possibility of repeater jamming. The implicit assumption that the jamming is a stationary process during each basic time interval eliminates swept-frequency jamming and deception jamming from consideration. The radar's options have been limited to the selection of a transmission frequency. If any of these restrictions on the strategies are removed, the game becomes difficult, if not impossible, to solve.

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[4] *D. J. White, Fundamentals of Decision Theory, American Elsevier, New York (1976).*

The preceding game, which was originally suggested by Nilsson,[5] is an illustration of a finite two-person zero-sum game. Infinite two-person zero-sum games that describe certain problems in electronic warfare have been solved in a few cases.[5,6]

4. RESOURCE ALLOCATION FOR AIRCRAFT ATTACK

A classic example of the application of game theory and optimization principles to electronic warfare arises in the planning of bomber penetration or an aircraft attack against an air defense radar system. The attacker must assign the roles of the available aircraft and allocate the jamming equipment and other resources. Some or all of the strike aircraft (or bombers) may be allocated jamming equipment. As "self-screening" jammers, strike aircraft can hinder both the ranging and the direction finding of a radar. However, the radiated energy may help the radar to locate the strike aircraft. The weight and the volume of the jamming equipment may necessitate a reduction in aircraft armament. Another tactic is to provide jamming equipment to auxiliary aircraft, termed "escort" jammers, that accompany some or all strike aircraft in close formation during part of their flights. Although the strike aircraft may be protected, the escort jammers may be susceptible to attack. A third possible tactic is to deploy jamming aircraft outside the detection range of the air defense system. Unless the jamming energy transmitted by these circling aircraft, which are termed "stand-off" jammers, is carefully coordinated with the flight patterns of the strike aircraft, the jamming energy usually penetrates through the sidelobes of the radar radiation pattern. Thus, high-power antennas are often required for the stand-off jammers to be effective. However,

[5] *N. J. Nilsson, An Application of the Theory of Games to Radar Reception Problems, IRE National Convention Record, 7, Part 4 (1959), 130-140.*

[6] *W. L. Root, Communications through Unspecified Additive Noise, Information and Control, 4 (1961), 15-29.*

it is possible for a diving aircraft to follow the main beam of a stand-off jammer, forcing the radar main beam to receive the jamming energy. The three jamming tactics are illustrated in figure 2.

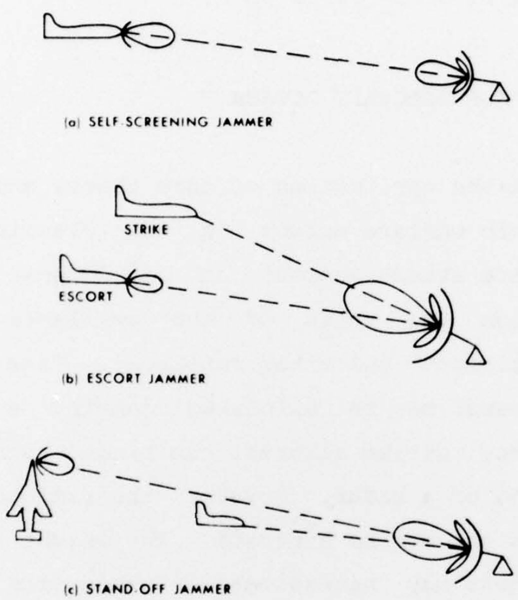

Figure 2. Radar jammers during aircraft attack.

Game theory is appropriate to the analysis of this conflict since evolving technology and tactics as well as incomplete intelligence and reconnaissance preclude a precise knowledge of the details of the air defense system. Each possible assignment of aircraft roles is labeled by an index i, while each possible type of air defense system is assigned an index j. Other parameters, denoted by the vector $\underline{x}$, must be chosen by the attacker. These parameters describe the characteristics of the jamming equipment and the weapons allocated to the aircraft. Each element of $\underline{x}$ is assumed to have a continuous or an infinite range of possible values because any element with a finite range is removed from the vector and used to enlarge the indexed set of the attacker's options.

To define a payoff matrix, we first define a function, represented by $f(i,j,\underline{x})$, which is the expected value of the measure of effectiveness of the mission; that is,

$$f(i,j,\underline{x}) = E\left[g(i,j,\underline{x})\right] . \tag{24}$$

For each value of $(i,j,\underline{x})$, $g(i,j,\underline{x})$ is a random variable on the sample space of mission outcomes. For example, $g(i,j,\underline{x})$ could be the gain from the complete or partial accomplishment of the attacker's mission minus the loss from the destruction of some of the attacking aircraft during the execution of the mission. The units of this function might be monetary or some measure of lost fighting capability.

An immediate difficulty is the problem of assigning a value to the loss of pilots. Such an assignment is highly subjective, if not impossible. A way out of this difficulty is to calculate the probability of pilot death or capture for each choice of i, j, and $\underline{x}$. Denoting this probability by $z_k(i,j,\underline{x})$, we impose the constraints that

$$z_k(i,j,\underline{x}) \leq C , \quad k = 1, 2, \ldots , \tag{25}$$

where C is the maximum acceptable value of this probability. The subscript k is used to distinguish the various types of aircraft (strike, escort, stand-off, etc.) that the pilots may fly.

If there are no possible choices of $\underline{x}$ that satisfy these inequalities for fixed values of i and j, then the option corresponding to index i is removed from consideration by the attacker. In other words, this option is no longer considered viable, regardless of its destructive potential. If no possible values of $\underline{x}$ and i can be found to satisfy equation (25) for each possible value of j, then either the mission must be cancelled or the value of C must be raised.

Assuming that equation (25) can be satisfied for some possible choices of $\underline{x}$, the attacker should choose $\underline{x}$ to maximize $f(i,j,\underline{x})$. Since the possible values of $\underline{x}$ indicate the characteristics of the jamming equipment and weapons aboard the aircraft, there are additional constraints to be considered. The most obvious constraints involve the dimensions and the weight of individual items or groups of items. Labelling each constraint by an index n, the constraints can be expressed symbolically by

$$y_n(i,\underline{x}) \leq D_n \; , \quad n = 1, 2, \ldots \; , \tag{26}$$

where the D_n are constants. Note that the functions $y_n(i,\underline{x})$ usually do not depend upon the index j. The maximization of $f(i,j,\underline{x})$, subject to equations (25) and (26), is accomplished by means of mathematical programming. We obtain a specific value of $\underline{x}$, denoted by $\underline{x}_0(i,j)$, which is the optimum value when i and j are specified.

We can now define an appropriate payoff matrix. The element values are given by

$$A_{ij} = f\left[i,j,\underline{x}_0(i,j)\right] \; . \tag{27}$$

Once this payoff matrix has been constructed, the optimal strategies can be evaluated according to the principles of the theory of zero-sum, two-person, finite games, for which effective computer algorithms exist.

A slightly different formulation of our resource allocation problem results if we have sufficient confidence in our intelligence and reconnaissance information to assign probabilities p_j to each defense option. We then define the expected values:

$$\overline{f}(i,\underline{x}) = \sum_j p_j f(i,j,\underline{x}) \; , \tag{28}$$

and

$$\bar{z}_k(i,\underline{x}) = \sum_j \hat{p}_j^{*} z_k(i,j,\underline{x}) \; . \tag{29}$$

For each fixed value of i, we choose $\underline{x} = \underline{x}_0(i)$ as the value of $\underline{x}$ that maximizes $\bar{f}(i,\underline{x})$ subject to the constraints given by equations (26) and

$$\bar{z}_k(i,\underline{x}) \leq C \; , \quad k = 1, 2, \ldots \; . \tag{30}$$

If no value of $\underline{x}$ satisfies these inequalities for some fixed i, then strategy i is eliminated from further consideration. Once $\underline{x}_0(i)$ has been determined, we define a payoff function by

$$f_1(i) = \bar{f}\left[i,\underline{x}_0(i)\right] \; . \tag{31}$$

The strategy i that maximizes the payoff function is the optimal one. An attractive feature of this formulation of the problem is that it automatically yields a pure strategy. The previous formulation with a payoff matrix yields a pure strategy only if the matrix contains a saddle point. If a saddle point does not exist, the optimal strategy is a randomized one. Unfortunately, when a single strike or a small number of strikes are launched, randomized strategies usually cannot be properly executed.

We now consider an extremely simplified and artificial example designed to illustrate the methodology of resource allocation for an aircraft attack. Suppose there are two possible air defense systems labeled by j = 1 and j = 2. Only two possible aircraft attack configurations are considered. If i = 1, a strike aircraft carrying self-screening jamming equipment is supported by a stand-off jammer. If i = 2, a strike aircraft without self-screening equipment is accompanied

by an escort jammer. The strike aircraft is labeled by $k = 1$, while the auxiliary aircraft is labeled by $k = 2$. The vector $\underline{x}$ is assumed to consist of a single element, denoted by w, which represents the weight of the jamming equipment on board the strike aircraft.

We assume that both the weight and the volume, v, of the jamming equipment are constrained. The constraints, which are symbolically represented in equation (26), are

$$0 \leq w \leq W \; , \tag{32}$$

and

$$0 \leq v \leq V \; , \tag{33}$$

where W and V are specified constants. We assume that

$$w = \rho v \; , \tag{34}$$

where ρ is a constant density. Combining these relations, we obtain

$$0 \leq w \leq \min(W, \rho V) \; , \tag{35}$$

which denotes the range of possible values for w.

By definition, the strike aircraft has no jamming equipment when $i = 2$. Thus, we may write $z_k(2,j,w) = z_k(2,j)$. We assume that, for the specified value of C,

$$z_k(2,j) \leq C \; , \quad j = 1, 2 \; , \quad k = 1, 2 \; , \tag{36}$$

where we recall that the left side is the probability of pilot loss for various types of aircraft (strike or escort in this case) and defenses.

When $i = 1$, we assume that the stand-off jammer is relatively safer from attack than the strike aircraft, so that

$$z_2(1,j,w) < z_1(1,j,w) , \quad j = 1, 2 . \tag{37}$$

This relation implies that $z_2(1,j,w)$ satisfies equation (25) if $z_1(1,j,w)$ does. The latter function is assumed to be linear, that is,

$$z_1(1,j,w) = a_j + b_j w , \quad j = 1, 2 . \tag{38}$$

Using these functional forms in equation (25) and combining the result with equation (35), we obtain the overall constraint,

$$0 \le w \le w_0 , \tag{39}$$

where

$$w_0 = \min\left(W, \rho V, \frac{C - a_1}{b_1}, \frac{C - a_2}{b_2}\right). \tag{40}$$

It is assumed that $w_0 \geq 0$ so that the constraint of equation (39) can be satisfied by at least one value of w.

Having examined all the contraints, we turn to the maximization of $f(i,j,w)$ for $i = 1, 2$ and $j = 1, 2$. Since the strike aircraft has no jamming equipment when $i = 2$, we may write $f(2,j,w) = f(2,j)$ for $j = 1, 2$. Suppose that $f(1,1,w)$ and $f(1,2,w)$ are maximized by $w = w_1$ and $w = w_2$, respectively, and that both w_1 and w_2 fall within the range specified by equation (39). Using equation (27), we may construct the payoff matrix:

Defensive strategies

Offensive strategies	$f(1,1,w_1)$	$f(1,2,w_2)$
	$f(2,1)$	$f(2,2)$

If it turns out that

$$f(1,1,w_1) < f(2,1) < f(2,2) \, , \tag{41}$$

then this game has a saddle point, and the optimal offensive strategy is to send an escort jammer along with the strike aircraft. In this case, the value of $f(1,2,w_2)$ is irrelevant, so we need not bother to determine w_2. If $f(1,1,w)$ is a linear function of w, then w_1 is either 0 or w_0.

Suppose a probability p_1 can be assigned to defense option j = 1. Then defense option j = 2 has a probability $p_2 = 1 - p_1$. Equations (29) and (36) imply that equation (30) is satisfied for k = 1, i = 2 and k = 2, i = 2. For simplicity, we assume that the stand-off jammer cannot be attacked when i = 1. Thus,

$$z_2(1,j,w) = 0 \, , \quad j = 1, 2 \, , \tag{42}$$

and equation (30) is satisfied for k = 2, i = 1. From equations (29) and (38), we have

$$\overline{z}_1(1,w) = p_1 a_1 + (1 - p_1)a_2 + \left[p_1 b_1 + (1 - p_1)b_2\right]w \, . \tag{43}$$

From equations (30), (35), and (43), we obtain the overall constraint,

$$0 \leq w \leq w_0' \, , \tag{44}$$

where

$$w_0' = \min \left[W,\ \rho V,\ \frac{C - p_1 a_1 - \left(1 - p_1\right) a_2}{p_1 b_1 + \left(1 - p_1\right) b_2} \right] . \qquad (45)$$

It is assumed that $w_0' \geq 0$, so that the constraint of equation (44) can be satisfied by at least one value of w.

From equation (28), we have

$$\overline{f}(1,w) = p_1 f(1,1,w) + \left(1 - p_1\right) f(1,2,w) . \qquad (46)$$

Suppose that this function is maximized by $w = w_1'$. Then equation (31) yields

$$f_1(1) = p_1 f\left(1,1,w_1'\right) + \left(1 - p_1\right) f\left(1,2,w_1'\right) . \qquad (47)$$

If $\overline{f}(1,w)$ is a linear function of w, then w_1' is either zero or w_0'. Using equations (28) and (31), we obtain

$$f_1(2) = p_1 f(2,1) + \left(1 - p_1\right) f(2,2). \qquad (48)$$

If it turns out that equation (41) is valid and

$$f\left(1,2,w_1'\right) \leq f\left(1,1,w_1'\right) , \quad w_1' = w_1 , \qquad (49)$$

then equations (41) and (47) to (49) imply that

$$f_1(2) > f_1(1) . \qquad (50)$$

Thus, the optimal offensive strategy is to send an escort jammer along with the strike aircraft.

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